

Active control of lateral balance in human walking

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Abstract

We measured variability of foot placement during gait to test whether lateral balance must be actively controlled against dynamic instability. The hypothesis was developed using a simple dynamical model that can walk down a slight incline with a periodic gait resembling that of humans. This gait is entirely passive except that it requires active control for a single unstable mode, confined mainly to lateral motion. An especially efficient means of controlling this instability is to adjust lateral foot placement. We hypothesized that similar active feedback control is performed by humans, with fore-aft dynamics stabilized either passively or by very low-level control. The model predicts that uncertainty within the active feedback loop should result in variability in foot placement that is larger laterally than fore-aft. In addition, loss of sensory information such as by closing the eyes should result in larger increases in lateral variability. The control model also predicts a slight coupling between step width and length. We tested 15 young normal human subjects and found that lateral variability was 79% larger than fore-aft variability with eyes open, and a larger increase in lateral variability (53% vs. 21%) with eyes closed, consistent with the model's predictions. We also found that the coupling between lateral and fore-aft foot placements was consistent with a value of 0.13 predicted by the control model. Our results imply that humans may harness passive dynamic properties of the limbs in the sagittal plane, but must provide significant active control in order to stabilize lateral motion. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Gait; Locomotion; Feedback; Sensorimotor control

1. Introduction

Although the biomechanical analysis of walking is quite complex, the fact that spinalized animals can produce a stepping motion that resembles normal gait implies that the generation of a stepping pattern may in some ways be simple (Grillner and Wallén, 1985). Nevertheless, these same animals do require external stabilization, suggesting that control of balance during gait is a more complex issue than is generation of the stepping pattern.

Walking differs from standing balance in that the center of mass constantly moves beyond the base of support, and in fact the support leg can do little to alter this motion (Winter, 1991). Townsend (1985) demonstrated that the control of placement of the swing foot can be used to stabilize balance in gait by manipulating the redirection of the center of mass that occurs as the

foot contacts the ground. Others (e.g., Winter, 1992; MacKinnon and Winter, 1993; Redfern and Schumann, 1994) have found empirical evidence for such control, indicating that balance during gait could be interpreted as an end-point control task.

It is, however, unclear how much neural control is needed, because the limb dynamics may already have stabilizing properties. McGeer's (1990a) study of passive dynamic walking demonstrated that in the plane of progression, the dynamics of two legs can interact such that they establish a periodic gait down a slight incline, with no need for external input except for gravity. He further showed that this limit cycle — periodic motion of a non-linear system — is also passively stable, meaning that small perturbations to the limit cycle are automatically dissipated so that the motion is gradually attracted back to the nominal gait pattern (see also Garcia et al., 1998). This finding suggests that except for supplying energy and supporting body weight, there may be little need for neural end-point control, because the planar passive dynamics alone are sufficient to provide the proper foot placement.

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We extended passive walking analysis to three dimensions, and found that although passive stability was retained in the plane of progression, there is a lateral instability that does require active control (Kuo, 1999). If these dynamics are representative of human walking, high-level neural feedback control would be necessary for maintaining lateral but not fore-aft stability. In the plane of progression, the support of body weight, supply of energy, and local stability that are available passively in the model could be performed by somatosensory feedback to the spinal cord (Grillner and Wallén, 1985). Weight support must also be provided in the lateral plane (MacKinnon and Winter, 1993), but the instability must be controlled by feedback. Proper sensing of lateral motion is likely to involve visual and vestibular input, which have been shown to be important in bipedal walking (Warren, 1998; Winter, 1995). Thus, the active control necessary in the model would be performed by higher centers such as the brain stem and cerebellum (Forsberg, 1985), based on integrated input from visual, vestibular, proprioceptive, and other sensors. Our model therefore suggests the biological hypothesis that active control from higher centers is necessary for lateral stabilization of gait, but the limbs and spinal cord are sufficient to provide other “passively” stable properties to the fore-aft motion.

2. Methods

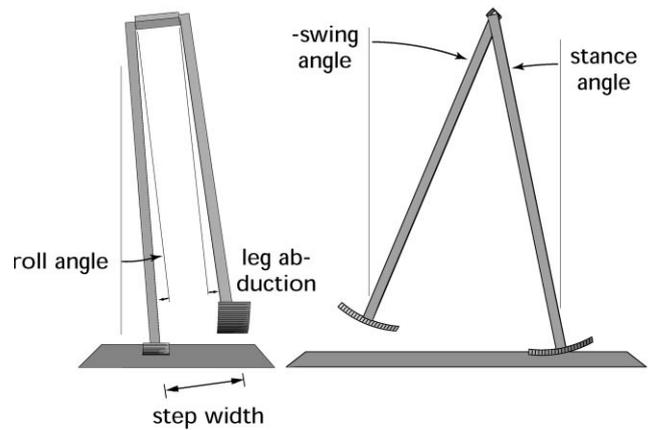
We devised a simple experiment to test the lateral balance control hypothesis by examining foot placement during unrestrained gait. One implication of our control model is that, under the influence of noise, lateral foot placement should exhibit greater variability than fore-aft placement. The loss of sensory input such as by removing vision should magnify uncertainty, resulting in still greater lateral variability but little effect on fore-aft variability. We measured variability of free gait in normal human subjects and compared them with the predictions made by our model.

2.1. Model

We review briefly the model of passive dynamic walking with lateral motion (Kuo, 1999; see webpage of the Journal of Biomechanics: <http://www.elsevier.nl:80/inca/publications/store/3/2/1/>).

The model consists of two legs connected by a pelvis, with pin joints at the hips. At the base of the legs are curved feet formed from sections of cylinders (see Fig. 1a). Pin joints at the feet allow the model to rotate in the frontal plane, with the amount of rotation referred to as the roll angle. The states of the equations of motion are described by the vector x , consisting of the roll, stance leg, and swing leg angles, as well as their respective time derivatives, or rates.

a. Passive dynamic walking model



b. Gait cycle simulation

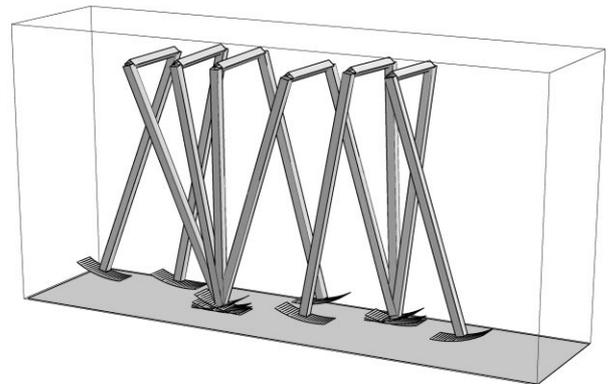


Fig. 1. Model of 3-D passive dynamic walking. (a) Three degrees of freedom describe motion of swing and stance legs in the sagittal plane, and the roll angle in the frontal plane. Leg abduction is adjusted quasi-statically to control step width. (b) Passive dynamics allow model to descend a gentle slope without external power, but lateral control is needed to withstand perturbations. Shown is nominal long-period gait for one stride. Additional detail regarding simulations is available at the web page of the Journal of Biomechanics: <http://www.elsevier.nl:80/inca/publications/store/3/2/1/>.

A step in the gait cycle consists of a full swing phase followed by an instantaneous transfer of support. In periodic motion, the following step is the same as its predecessor except that the legs switch roles. The trajectory of the swing phase is found by numerically integrating the nonlinear dynamical equations of motion over time until the swing foot contacts the ground. The contact is modeled as an instantaneous, perfectly inelastic impact that creates the initial conditions for the next step. This entails changes in the roll, stance, and swing rates that are computed using three equations for conservation of angular momentum. After ground contact, the legs are relabeled so that the previous stance and swing legs are swapped. Combining all of these procedures yields the step-to-step function

$$x_{k+1} = F(x_k), \quad (1)$$

that takes as input the initial state vector x for step k and returns the post-support-transfer initial conditions for step $k + 1$.

For gait, two concerns with regard to equation (1) are the existence of a periodic gait cycle, and whether it is stable if it does exist. A periodic gait or limit cycle exists if there is a steady-state x_{ss} such that

$$x_{ss} = F(x_{ss}). \tag{2}$$

There are in fact two solutions, a short- and a long-period gait, of which the long-period cycle bears a greater resemblance to human walking (Fig. 1b). Concentrating on this latter solution, the local asymptotic stability is assessed by evaluating the eigenvalues of the Jacobian function

$$A \equiv \left. \frac{\partial F}{\partial x} \right|_{x_{ss}}.$$

If the eigenvalues have magnitude less than one, the limit cycle is locally stable and will dissipate small perturbations. The eigenvectors describe modes along which perturbations are decoupled from each other.

Stability analysis shows that the stability of the fore-aft modes is retained from the planar case, but one mode that is primarily of lateral motion is unstable. Fortunately, this mode has only a slight coupling to the fore-aft states, and is easily stabilized with lateral foot placement with little effect on fore-aft motion (see Fig. 2). We designed a negative feedback law to quasi-statically alter the splay angle of the legs by an amount

$$u_k = -K(x_k - x_{ss}), \tag{3}$$

where u_k is the adjustment to lateral placement for the following step and K is the feedback gain, which is dominated by terms for roll angle and rate, with smaller terms related to stance angle and rate (see the appendix). This control law drives a perturbation to zero by the end of the next step.

Further analysis shows that lateral foot placement can be achieved at low-energy cost compared to other methods of stabilization. Ankle inversion/eversion (Matsusaka, 1986) and hip ab-/adduction (Winter et al., 1996) are also possible methods, but provide limited control authority (Kuo, 1999). The relatively light coupling between the lateral and fore-aft dynamics implies that they can be considered separately (see Fig. 3). The passively stable mechanics of fore-aft motion can be provided by the human legs with relatively simple reflex action involving proprioceptors and the spinal cord to provide energy input and weight support. In contrast, the lateral motion requires an active feedback loop, most likely involving visual, vestibular, and other sensors. Sensor noise should contribute to variability in the lateral mode and therefore in lateral foot placement, but should have little effect on the fore-aft modes or foot placement.

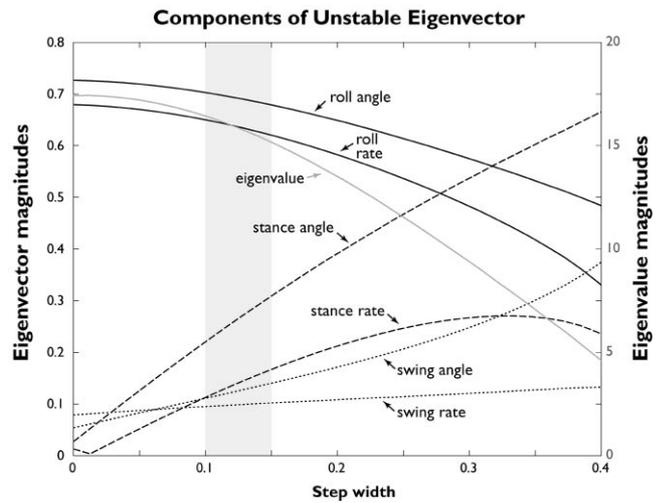


Fig. 2. Passive dynamic walking has one unstable mode, which is largely confined to frontal plane. Magnitudes of unstable eigenvector components are shown vs. step width (normalized by leg length). For small step widths, roll angle and rate dominate, so that fore-aft motion is largely unaffected. Shaded region denotes typical step widths chosen by normal subjects. Also shown is magnitude of unstable eigenvalue in gray, with axis to right. Note that instability decreases with step width.

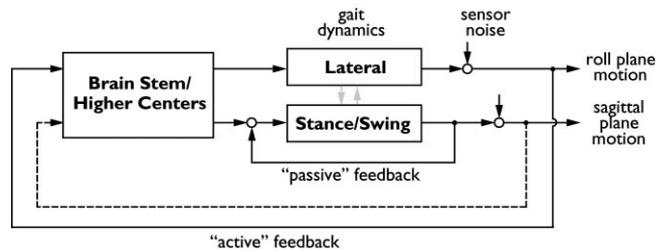


Fig. 3. Hypothetical feedback control diagram for walking. In passive walking model, lateral instability is largely decoupled from dynamics of swing and stance legs in the sagittal plane (gray arrows denote weak coupling). Fore-aft dynamics are passively stable and lateral dynamics require active control. In humans, both limb dynamics and spinal cord presumably provide the equivalent of “passive” stabilization, including power and weight support. Lateral stabilization is likely to involve visual and vestibular sensors in a feedback loop (brain stem and higher centers), and is therefore sensitive to sensor noise. Fore-aft dynamics do not require such feedback (dashed line), and is therefore likely to be less sensitive to sensor noise.

Furthermore, reduction of visual input reduces the amount of sensory information available (equivalent to increasing sensor noise), which should have a greater impact on lateral than fore-aft foot placement. A more detailed description of this reasoning is given in the appendix.

Our hypothesis of active control of lateral balance is tested by comparing lateral and fore-aft foot placement. We first expect that the lateral variability of foot placement should be larger than the fore-aft variability, assuming that intrinsic noise-like perturbations are not

grossly unequal for the two planes of motion. Second, we expect that lateral variability should increase with loss of vision, while fore-aft variability should be affected much less. Third, we expect a slight correlation between lateral and fore-aft foot placement due to a coupling term in the control gain (Eq. (3)).

3. Experimental measurements

To test our hypothesis regarding control of lateral balance, we measured lateral and fore-aft foot placement in gait with eyes open and with eyes closed. Our protocol made it desirable to provide a normal visual input during the eyes open trials, and to measure gait over a sufficient number of contiguous steps to provide consistent statistical measures of foot placement variability. Because both a treadmill and a fixed gait lab with a limited sensing volume were deemed unsuitable for these purposes, we developed a mobile measurement system to record free gait over an unlimited sensing volume.

Fifteen normal young adult subjects (four females and 11 males, 21–37 yrs), suffering from no known gait abnormalities, participated in the study. All subjects provided informed consent. Average height was 176.5 cm and average body mass was 72.2 kg.

Each subject was asked to walk in a straight line, marked by a set of traffic cones, at a freely chosen speed. Data were collected in trials, each at least 100 steps in length. Each condition was applied four times, providing at least 400 steps of data. The conditions consisted of eyes open (EO) and eyes closed (EC). During the EC trials, the subjects were asked to follow an audible cue to allow them to follow an approximately straight path. The audible cue was provided by a member of the investigation team walking directly ahead of the subject and carrying a portable tape machine playing music. Some subjects found EC to be disconcerting at first, but typically became comfortable after 1–2 min of practice.

Kinematics were measured using a mobile measurement system based on a magnetic tracking system (Ascension Technologies Corporation, VT). This system provides six-degrees-of-freedom position and orientation information for eight magnetic receivers attached to the subject's body. The receivers were mounted on the top of each foot (above the first and second metatarsal heads), on the front of each shank (at the tibial tuberosity), on the front of each thigh (on the anterior surface, midway between the knee and hip), at the back of the pelvis, and at the back of the head. The sampling rate was set to 100 Hz. Although all markers were used to ensure that the measured gait appeared to be normal, only the foot location data will be reported here.

In order to measure kinematics during many contiguous steps with a normal visual input, we mounted the tracking system on two rolling carts. The first cart carried

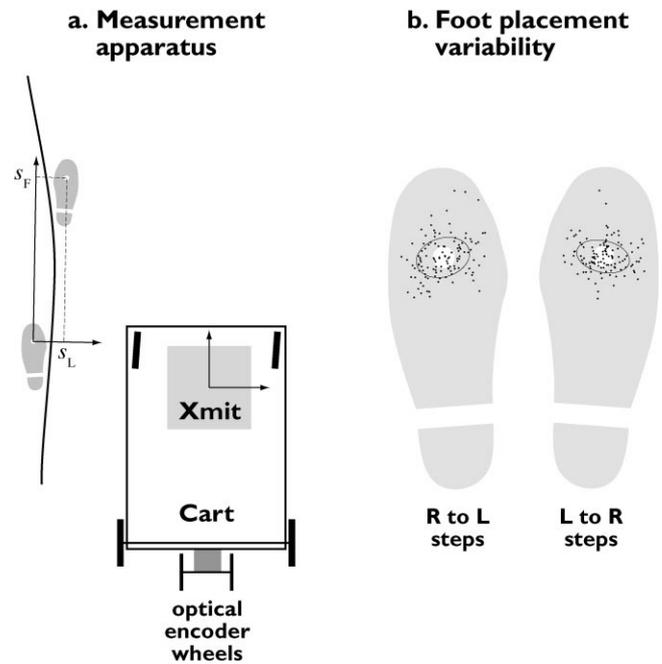


Fig. 4. (a) Kinematic measurement of foot placement, as viewed from above. A motion tracking system consists of a magnetic transmitter (Xmit) mounted on a rolling cart and pushed near the walking subject, who has magnetic receivers attached to the limbs. This system measures motion of the limbs relative to the cart, and optical encoders measure motion of the cart relative to ground. Systematic measures include step length s_F and step width s_L . (b) Foot placement varies relative to nominal length and width, as shown relative to scale of foot. Black dots mark each of approximately 100 individual steps, and white circles denote approximate location of magnetic receivers. Ellipse denotes the $1 - \sigma$ covariance, and the tilt of this covariance indicates slight coupling between lateral and fore-aft foot placement, as predicted by the control model.

the magnetic transmitter and was pushed directly behind or to the side of the subject at a distance of less than 1 m (see Fig. 4a). Optical encoders were mounted on the rear wheels of this cart to provide information about the translation of the tracking system's coordinate frame with respect to earth. The second cart carried data collection equipment and power supplies, and was pushed at a distance of 2–3 m behind the first cart in order to minimize interference with the magnetic field.

Kinematic data were filtered using a digital third-order low-pass Butterworth filter with a cutoff frequency of 5 Hz, applied in forward and backward directions to remove transient artifacts.

From the relative position information provided by the receivers mounted on the feet, we identified a consistent point in the step cycle for use in computing the relevant gait parameters. This point was found by first identifying as ground contact intervals the instances during which a foot was simultaneously at a constant vertical position and moving backwards with respect to the tracking system's coordinate frame. From these intervals,

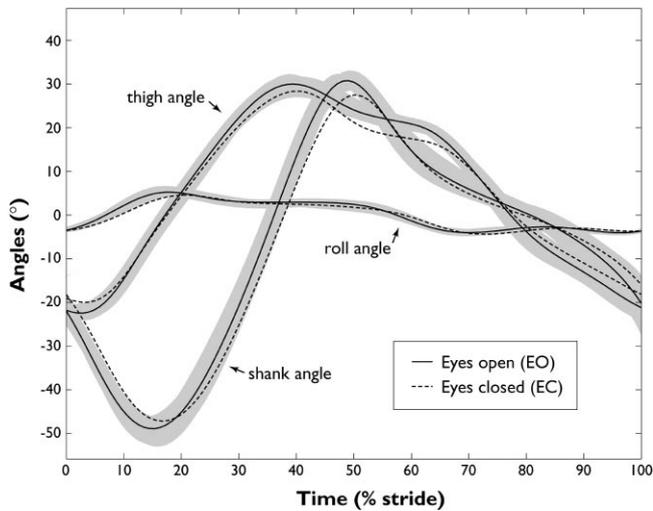


Fig. 5. Comparison of limb kinematics for one representative subject, eyes open (EO) vs. eyes closed (EC). Solid black line shows mean trajectories of segment angles, and gray region shows full range of trajectories over one EO trial. Dashed black line shows mean trajectories for EC. Except for slightly decreased step length, subjects walked with similar kinematics for EC.

our algorithm selected the sample prior to the foot leaving the ground. The period between these points was very consistent, typically varying by two sample points or less over an entire trial.

Absolute position information was found by adding translation of the tracking system's coordinate frame to the relative information. The position of this frame was found by reversing the relative translation of the foot during ground contact periods. For intervals when both feet were on the ground, the data from the front foot was used. This absolute trajectory was verified against optical encoder data and then used to compute a direction of travel using a moving window formed from four steps. The lateral and fore-aft directions were defined, respectively, as perpendicular to and parallel with this direction of travel, in order to permit subjects to deviate slightly from a perfectly straight path while maintaining a locally meaningful frame of reference (Fig. 4b).

The first parameters computed from the kinematic data were systematic parameters such as the average walking speed and cadence, and the step width and step length (see Fig. 4a). These latter parameters were defined to be equal to the average displacement of each step in the lateral and fore-aft directions, s_L and s_F , respectively. We then computed the step variability using the standard deviation steps in the lateral and fore-aft directions, σ_L and σ_F , respectively. All step length and variability measures were normalized by the leg length, L , of each subject.

We used repeated measures analysis of variance (ANOVA) on these gait parameters to test for significant differences between lateral and fore-aft variability, be-

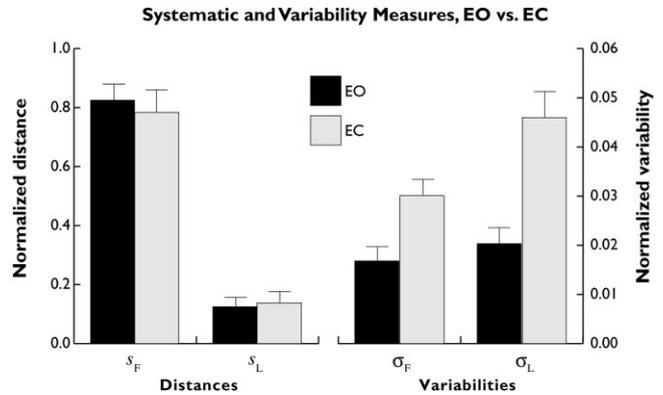


Fig. 6. Average systematic and variability measures of foot placement for normal subjects, eyes open (EO) vs. eyes closed (EC). Step length s_F decreased slightly and step width s_L increased slightly with EC. Lateral variability σ_L exceeded fore-aft variability σ_F in both EO and EC conditions. Changes in variability with EC far exceed changes in systematic measures. Shown are average values and error bars for standard deviation across 15 subjects.

tween EO and EC conditions, and to test for interaction between the two. When statistically significant differences were found, paired t -tests were also used to compare differences within factor (see Fig. 5).

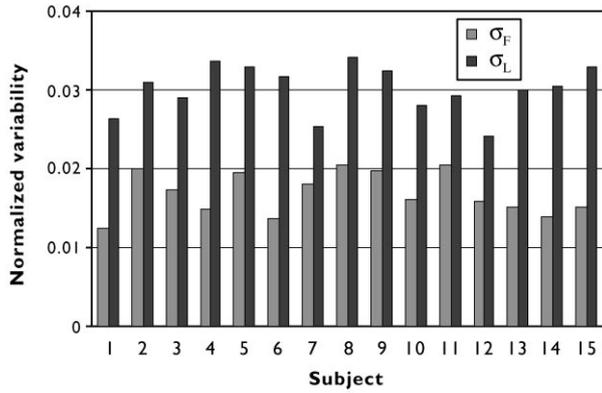
The final measurement was of coupling between fore-aft and lateral foot placement. We evaluated the slope of the major axis of the ellipse describing covariance of foot placement (see Fig. 4b), which is directly comparable to the step width/length gain predicted to be 0.13 (see the appendix). We compared this prediction against the 95% confidence interval for the empirically-derived mean step width/length gain.

4. Results

We report results in two categories: systematic gait parameters, and variability parameters. The latter parameters were the primary focus for testing our hypotheses, but the former measures are useful for insuring that any changes in variability were not merely the consequence of alterations to the regular gait pattern.

Systematic gait parameters were typical of normal gait patterns and differed little between EO and EC conditions (see Fig. 6). In the EO condition, subjects walked with an average cadence of 117 ± 6.9 steps/min. (mean \pm standard deviation), speed of 1.5 ± 0.11 m/s, and stride length, defined as two consecutive steps, of 1.57 ± 0.04 m. In the EC condition, subjects walked at a slightly lower cadence of 115.4 ± 7.3 , lower speed of 1.41 ± 0.15 m/s, and lower stride length of 1.50 ± 0.04 m. Comparing EC to EO, step length s_F decreased by an average of 5.1%, while step width s_L increased by 11%. These differences were statistically significant ($p < 0.01$),

a. Variability EO



b. Change in Variability EO to EC

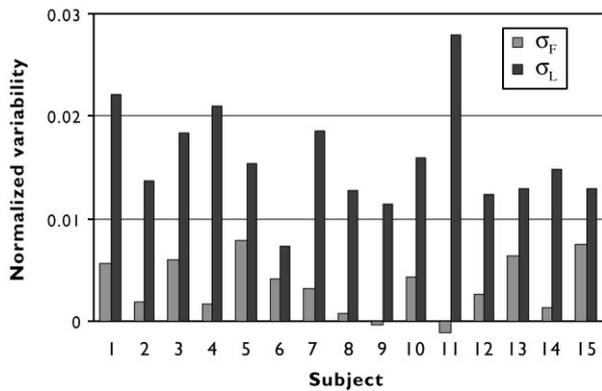


Fig. 7. Measures of foot placement variability for all subjects. (a) All subjects had greater lateral variability (σ_L) than fore-aft (σ_F) variability with eyes open. (b) Change in lateral variability with EC exceeded change in fore-aft variability, in both absolute and relative terms, in all subjects.

but were small in comparison to those seen in variability measures.

Consistent with predictions, lateral variability exceeded fore-aft variability, and EC vs. EO had an especially large effect on lateral variability. ANOVA revealed significant differences for lateral vs. fore-aft variability ($p = 5.3E - 28$) and EO vs. EC conditions ($p = 1.4E - 14$), as well as interaction between factors ($p = 1.4E - 8$). Within the EO condition, lateral variability σ_L was 79% larger than fore-aft variability σ_F (see Fig. 6) with high significance in the paired t-test ($p = 2.6E - 18$), with average values of $\sigma_L = 0.0301 \pm 0.0033$, $\sigma_F = 0.0168 \pm 0.0029$. Within the EC condition, σ_L exceeded σ_F by an even greater amount, 126% ($p = 2.3E - 20$). Average values for EC were $\sigma_L = 0.0460 \pm 0.0053$ and $\sigma_F = 0.0203 \pm 0.0033$. The increase in σ_L from EO to EC was 53% in contrast to an increase in σ_F of only 21% (see Fig. 7). The average increases were significantly different for the lateral and fore-aft directions ($p = 7.26E - 12$), 0.0159 ± 0.0052 for

Step width/length coupling

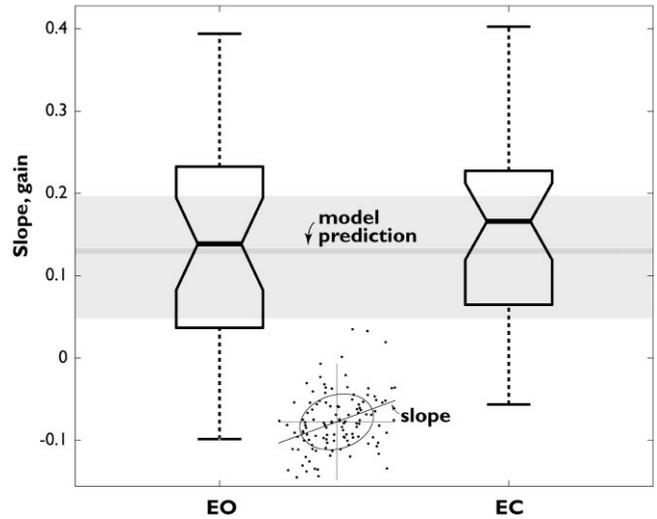


Fig. 8. Box plot of coupling between step width and length for EO and EC, compared against prediction from stabilizing feedback control. Vertical axis shows gain of step width to step length change. Plot shows mean (solid black line) within a box denoting the first and third quartile values. Notches denote the 95% confidence interval surrounding the mean. The model predicts that a control law should produce a gain of approximately 0.13 (gray line), with the range of possible stabilizing gains shown (light gray shaded region). Inset diagram shows a covariance ellipse for right-to-left steps (from Fig. 4b). The slope of the major axis is directly comparable to gain. For left-to-right steps, the ellipse is flipped horizontally to rectify the slope.

σ_L , and 0.0035 ± 0.0029 for σ_F . The experiment-wise error probability for these multiple comparisons was low ($p = 1.4E - 8$).

Subjects also exhibited coupling between step width and length consistent with predictions (see Fig. 8). The width/length gain was 0.134 for EO, and 0.156 for EC. The predicted value of 0.13 fell within the 95% confidence intervals for these data.

5. Discussion

Our results are fully consistent with the hypothesis that lateral balance must be stabilized with visual-vestibular feedback, but fore-aft dynamic stability requires little or no feedback of this type. The addition of visual-vestibular feedback that is necessary for stability implies a sensitivity to intrinsic perturbations and sensor noise, which should result in the larger variability in the lateral vs. fore-aft directions that was observed in the EO conditions. The reduction in sensory information with EC results in a clear increase in the lateral variability as expected. The relatively small change in fore-aft variability indicates a relatively low sensitivity, in normal above-ground conditions, to the amount of visual-vestibular information needed for fore-aft control.

None of these conclusions can be explained by systematic changes in the gait pattern between EO and EC conditions. The largest differences in systematic parameters were on average 11% increase in step width and 6.3% decrease in cadence with EC, but these were far exceeded by the increase in lateral variability, 53%. The model presents two possible advantages to increasing step width when lateral stability is made difficult. First, if a consistent lateral clearance of the swing foot from the stance foot is desired, the step width should increase proportionately to lateral variability. Second, there is a slight stability advantage with wider steps, because the unstable lateral eigenvalue decreases with width (Kuo, 1999).

The evidence for active control is strengthened by the slight coupling between lateral and fore-aft foot placement as predicted by the model. The step width/length gain (Fig. 8) is consistent with that needed to stabilize the model, and also appears to be responsible for the relatively small 21% increase in fore-aft variability σ_F accompanying the 53% increase in σ_L with EC. This coupling cannot be explained by the dynamics of the legs alone, which would predict a much larger gain (see the Appendix).

Although we have found a great deal of consistency between model and experimental results, it is important to caution that the model is not an exclusive explanation for our data. An alternative explanation might be that intrinsic perturbations and sensor noise (which are difficult to assess quantitatively) are merely biased in the lateral direction, but this is unlikely given the trends seen in the EC conditions. It is also possible that uncontrolled factors such as nervousness in the subjects, unequal noise or perturbation levels between fore-aft and lateral directions, could also produce similar results. A more sophisticated test would be to compare the model's closed-loop behavior with autoregressive characteristics extracted from the kinematic data. However, a truly definitive test of the lateral control hypothesis would require a means to open or manipulate the visual-vestibular feedback loop in vivo.

Other limitations are posed by the simplicity of the model, which currently lacks degrees of freedom such as an upper body and knees. McGeer's (1990a,b) studies indicate that such additions would not substantially affect model predictions regarding leg motion, although stabilization of the upper body would require active control (see also Warren et al., 1996). At present the model is also limited to forward walking and cannot accommodate turning.

The large differences in lateral variability associated with EC conditions suggest that this measurement may be useful as a quantitative assessment of sensorimotor control. Conventional gait measures concentrate on systematic changes to the gait pattern (McFadyen and Belanger, 1997), and are not typically suited for testing

sensorimotor control. Discrimination between EO and EC using lateral variability was both larger and more consistent than would appear possible using either subjective observations of the actual gait, or quantitative comparisons of systematic gait parameters such as step length and frequency or joint angle trajectories. Variability might, for example, be useful for assessing recovery or compensation of sensorimotor control following trauma or disease. Such a clinical application might, however, be more practical if implemented on a treadmill. A comparison of variability between above-ground and treadmill gait would be useful for establishing the latter as an assessment tool.

Our results demonstrate that the reduction of sensory information has a greater impact on lateral control of balance during gait, but local stability of the steady walking cycle is only a single component of a task with multiple concerns. Different effects would be expected with large perturbations or incorrect or illusory sensory information. Vision, for example, is also important for dealing with obstacles (Patla, 1998) and for controlling heading, speed, and upper body sway. Warren et al. (1996, 1998) found greater lateral vs. fore-aft body sway in subjects walking on a treadmill with a simulated visual display of a stationary hallway. However, in unusual conditions such as a traveling hallway moving perpendicular to the treadmill, oscillatory perturbations of the visual field could induce opposite effects. Even if "passive" stability to small perturbations can be provided by spinal or low-level control, larger perturbations and other concerns demand that additional integrative information from visual and other sensors also influence fore-aft motion of the legs during walking.

Acknowledgements

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Appendix

This section presents the lateral-control law and the associated predictions regarding variability of foot placement. It makes use of the linearized step-to-step equation (Kuo, 1999) with the addition of a control input for lateral foot placement and a perturbation input for noise-like influences:

$$\Delta x(k+1) = A\Delta x(k) + Bu(k) + \Gamma w(k), \quad (A1)$$

where $u(k)$ is the control input, $w(k)$ is the perturbation input, and B and Γ describe the influence of those two inputs, respectively. $\Delta x(k)$ refers to the deviation of the state from the nominal x_{ss} .

The stabilizing control law is

$$u(k) = -K(\Delta x(k) + v(k)), \quad (\text{A2})$$

where

$$K = \begin{bmatrix} -0.98 & 0.13 & -0.0063 & -0.98 & 0.16 & -0.0061 \end{bmatrix}. \quad (\text{A3})$$

(Kuo, 1999) and $v(k)$ describe measurement errors due to sensor noise. Assuming that both $w(k)$ and $v(k)$ are random variables with zero mean and Gaussian distribution, the statistical properties of (A1) can be propagated forward in time. Taking the covariance of (A1) with (A2) yields

$$X(k+1) = AX(k)A^T + BKX(k)K^TB^T + BKVK^TB^T + \Gamma W \Gamma^T, \quad (\text{A4})$$

where $X(k)$, $W(k)$, and $V(k)$ are the covariances of $\Delta x(k)$, $w(k)$, and $v(k)$, respectively.

The lateral control hypothesis arises from the partial decoupling of the lateral and fore-aft dynamics (Fig. 1). This implies that (A1) can be partitioned into two subsystems describing the lateral dynamics (denoted by subscript L) and fore-aft dynamics (denoted by subscript F). The lateral dynamics are unstable and include the control law, so that the covariance is

$$X_L(k+1) = A_L X_L(k)A_L^T + B_L K_L X_L(k)K_L^T B_L^T + B_L K_L V_L K_L^T B_L^T + \Gamma_L W_L \Gamma_L^T. \quad (\text{A5})$$

The fore-aft dynamics need not be controlled because they are passively stable, and so

$$X_F(k+1) = A_F X_F(k)A_F^T + \Gamma_F W_F \Gamma_F^T. \quad (\text{A6})$$

Foot placement is given by linear transformations of the lateral and fore-aft states. Noting that (A5) includes terms involving sensor noise that are not present in (A6), we expect the lateral covariance, and therefore foot placement, to be larger (EO condition). Because we have not devised an explicit model for perturbations $w(k)$, we must assume that Γ and $W(k)$ are approximately balanced between lateral and fore-aft states.

The EC condition however provides a means to control for the effect of perturbations and determine the influence of increased sensor noise. We assume that with EC, the amount of sensory information decreases, thereby increasing $V(k)$. This should result in an increase in X_L but not X_F unless there is substantial coupling to fore-aft states in the unstable mode. This increase should be reflected in an increase in lateral foot placement variability.

The control law also predicts a small degree of coupling between lateral and fore-aft foot placement. The second entry in K (A3) predicts a lateral adjustment in the amount of 0.13 for a unit change in stance angle. Because

these two quantities are approximately one-half of step width and length, respectively, it is expected that the change in step width should be 0.13 times the change in step length. Humans need not use the same control law as the model; however, investigation of the range of stabilizing gains reveals that they are restricted to a relatively narrow band surrounding 0.13 (see Fig. 8). This coupling is a prediction is a result of the control law rather than the passive dynamics alone. For example, the passive effect of a unit change in lateral foot placement on step length can be computed from previous results (Kuo, 1999) to predict the equivalent of a gain of 0.54 for step width/length.

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